

On the crucial role of higher order evanescent modes for double-tuning of same-end inlet-outlet and opposite-ends inlet-outlet elliptical chamber mufflers

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Abstract

Elliptical chamber mufflers are used where in one transverse direction less space is available as compared to the other transverse direction, e.g., automotive vehicle mufflers under the chassis. The inlet and outlet port locations are often logistic constraints. The simplest muffler configurations with same-end inlet-outlet (SEIO) and opposite-end inlet-outlet (OEIO) are popularly known as simple expansion chamber (SEC) and reversing chamber muffler (RCM), respectively. Both SEC and RCM are not suitable for automotive applications due to their narrow band transmission loss spectra with periodic domes of width of $kL = \pi$, where k is the wave number and L is the chamber length. However, by extending one of the inlet and outlet ports by $L/2$ acoustic length and other by $L/4$ acoustic length inside the chamber gives wide-band transmission loss spectra of SEIO and OEIO mufflers with periodic domes of width of $kL = 4\pi$. Moreover, the above mentioned extensions also result in the higher value of transmission loss for $kL \in [\pi/2, 7\pi/2]$. In order to satisfy the acoustic mass continuity and acoustic particle velocity compatibility at the junctions of the extended pipes, annular region surrounding the extended pipes and the muffler chamber, the higher order evanescent modes are generated. The effect of the higher order evanescent modes results in the inline inertance of the extended pipes, which makes the acoustic length of the extended pipes more than their geometric lengths. The difference between the acoustic length and the geometric length is known as the end-correction. In this study, the end-corrections are evaluated by 3-D finite element analysis. In the present study, the effect of different geometrical parameters on the end-corrections of elliptical chamber mufflers for double-tuning is investigated.

Keywords: Higher order evanescent modes, End-correction, Finite Element analysis, Elliptical Chamber, Double-tuning.

1. Introduction

Low centre of gravity of automotive vehicles results in less ground clearance available for the mufflers beneath the chassis. Therefore, the elliptical chamber mufflers are mostly preferred in automotive vehicles especially in four-wheelers. The simplest reactive muffler configurations depending on the same-end inlet-outlet and

opposite-end inlet-outlet are simple expansion chamber (SEC) and reversing chamber muffler (RCM), respectively [1]. Both SEC and RCM transmission loss (TL) spectra has periodic domes with periodicity of $kL = \pi$ in the plane wave region. Thus, the SEC and RCM mufflers are not better choices for the applications where the significant frequency range of unmuffled

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sound pressure level (SPL) spectrum varies with the operating conditions of the source, e. g. automotive vehicles.

The simplest improvisation on both SEC[2] and RCM[3] is to extend one of the inlet/outlet pipes inside the chamber by $L/2$ acoustic length and the other by $L/4$ acoustic length to have four times wider transmission loss spectra. The quarter-wave (Q-W) resonators of the annular region surrounding the extended pipe of length $L/2$ and $L/4$ lifts the troughs at odd multiples of $kL = \pi$ and odd multiples of $kL = 2\pi$, respectively, which is known as the double-tuning of axially long SEC and RCM. Moreover, above mentioned extensions of the inlet pipe and outlet pipe inside the chamber gives higher values of transmission loss for $kL \in [\pi/2, 7\pi/2]$ as compared with their counterpart muffler configurations without extensions of the inlet pipe and outlet pipe. For chamber area to the inlet/outlet pipe area ratio, $m \gg 1$, the envelope maximum value of the TL spectrum excluding the narrow peak at $kL = 2\pi$ of a double-tuned muffler configuration is approximately 12 dB more than the simple expansion chamber muffler[4]. Thus, the double-tuning not only helps in widening the TL spectrum band but also helps in lifting the TL spectrum.

The extended pipes of physical lengths of $L/2$ and $L/4$ does not result in the double-tuning because of the higher-order evanescent modes generated at the junctions of extended inlet/outlet pipes, chamber and the annular region surrounding the extended pipes. These higher-order evanescent waves get generated in the plane frequency region to satisfy the acoustic mass continuity and acoustic particle velocity compatibility at the junctions of the extended pipes, annular region surrounding the extended pipes and the muffler chamber. The end-corrections of the extended inlet/outlet pipes are the approximations of the effect of the higher

order evanescent waves generate at the junctions in the form of inline inertance of the extended pipes. Recently, the crucial role of the higher order evanescent modes generated at the junctions for the double-tuning of a rotated-offset opposite-end inlet-outlet circular chamber muffler was highlighted by Kumar *et al*[2]. Double-tuning of the same-end inlet-outlet (SEIO) circular chamber mufflers was done by Gaonkar *et al*[3].

For the double-tuning of an axially long circular chamber muffler, the plane wave analysis must hold at least up to $kL = 4\pi$. However, the cut-on frequency of the first possible higher-order mode depends on the location of the ports and the diameter of the chamber. Therefore, to have the clearly noticeable double-tuning the length to diameter ratio, $L/D \geq 2\pi/\alpha_{m,n}$ [2,5], where $\alpha_{m,n} = 3.83$ for the centre inlet-outlet and $\alpha_{m,n} = 1.84$ for the offset inlet-outlet. In the case of elliptical chamber mufflers with offset inlet-outlet, the first possible higher-order mode to get cut-on is $(1, 1)_e$ mode which gets cut-on at $0.5kD_1 = 1.86 \pm 2\%$, where D_1 is the major axis length of elliptical cross-section. For the same cross-sectional area of an ellipse and a circle, the major axis length D_1 is always greater than the circle diameter D , and more is the eccentricity of the ellipse more is the value of D_1 . Thus, the limit of the plane wave analysis is lesser for an elliptical chamber muffler as compared with a circular chamber muffler of the same cross-sectional area. Therefore, the aspect ratio, D_2/D_1 is an important factor which has effect on the end-corrections.

In the present study, the end-corrections of the extended inlet/outlet pipes are computed using finite element analysis for variation of different geometrical non-dimensional parameters.

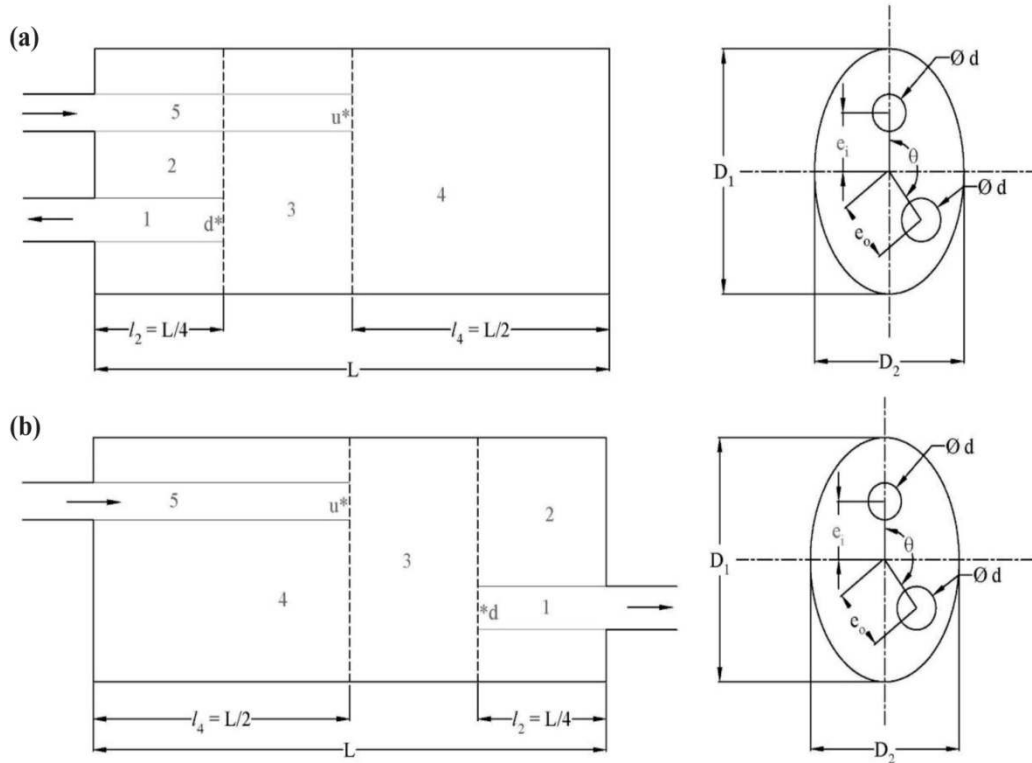


Fig.1 —Schematic diagram of the double-tuned elliptical chamber mufflers: (a) same-end inlet-outlet and (b) opposite-ends inlet-outlet

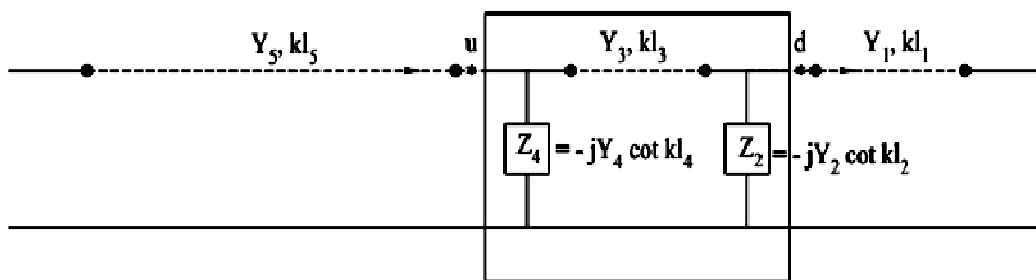


Fig.2 —Electro-acoustic circuit of the extended inlet/outlet mufflers shown in Fig. 1

2. Double-tuning of the SEIO and OEIO mufflers

The schematic diagram of the same-end inlet-outlet (SEIO) and opposite-ends inlet-outlet (OEIO) elliptical chamber mufflers are shown in Figs. 1(a) and 1 (b), respectively. The electro-acoustic circuit for both the mufflers are same which is shown in Fig. 2. The descriptions of all the geometrical parameters of elliptical chamber mufflers are given in Table 1.

Table 1—Description of all geometrical parameters of an elliptical chamber muffler

Symbol	Description
L	Length of the chamber
D_1	Major axis of the elliptical cross-section
D_2	Minor axis of the elliptical cross-section
d	Diameter of the inlet/outlet pipes
t_w	Inlet/outlet pipes wall thickness
e_i	Offset of the inlet pipe from the centre
e_o	Offset of the outlet pipe from the centre
θ	Angle between the inlet port and outlet port

The transmission loss spectra of tuned and untuned muffler configurations of SEIO with $\theta=180^\circ$, OEIO with $\theta=0^\circ$ and OEIO with $\theta=180^\circ$ are shown in Figs. 3, 4 and 5, respectively. To find out the end-corrections of the extended pipes of lengths $L/2$ and $L/4$, 3D finite element analysis (FEA) is carried out for the muffler and the shift in peak frequencies from their expected peak frequencies for corresponding double-tuned muffler configuration are calculated. The shift in peak frequencies are the results of inline inductance of extended pipes due to the generation of the higher-order evanescent modes at the junctions. Therefore, the first and second TL peaks

corresponding frequencies computed by 3D FEA are given by Eqs. 1 and 2. In Eq. 1, the negative sign is for SEIO muffler configurations because due to the end-correction of $L/2$ length extended pipe, $\delta_{0.5L}$, the length of the Q-W resonator is decreased.

$$f_{1p}^{3D} = \frac{c}{4(0.5L \pm \delta_{0.5L})} \quad (1)$$

$$f_{2p}^{3D} = \frac{c}{4(0.25L + \delta_{0.25L})} \quad (2)$$

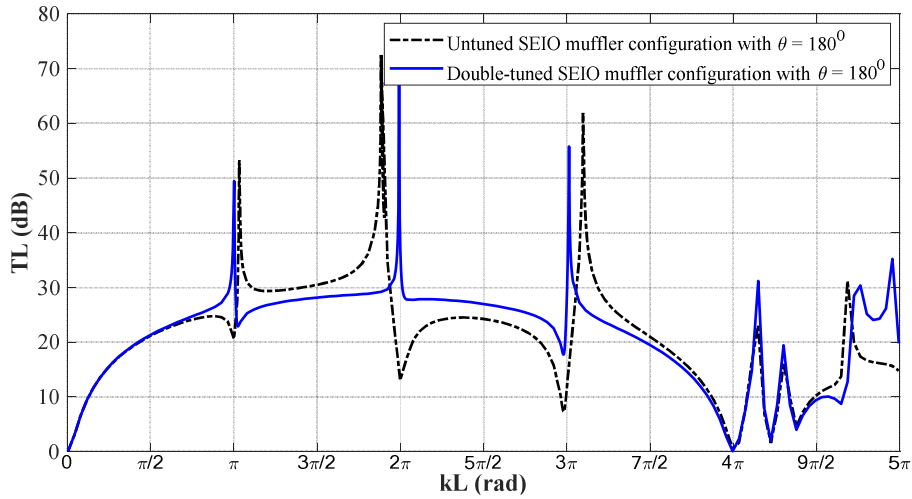


Fig. 3 — Comparison of the TL spectra of untuned and double-tuned SEIO muffler configurations with $\theta=180^\circ$, $d = 30 \text{ mm}$, $m = D_1 D_2 / d^2 = 16$, $AR = D_2 / D_1 = 0.4$, $e_i / D_1 = e_o / D_1 = 0.3$, $L / D_1 = 3.5$, and $t_w = 1.4 \text{ mm}$

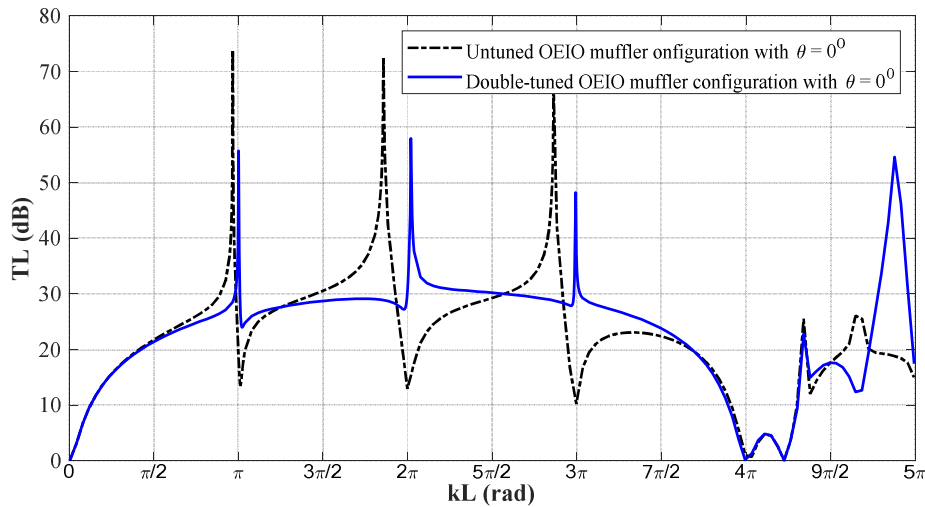


Fig. 4 — Comparison of the TL spectra of untuned and double-tuned OEIO muffler configurations with $\theta=180^\circ$, $d = 30 \text{ mm}$, $m = D_1 D_2 / d^2 = 16$, $AR = D_2 / D_1 = 0.4$, $e_i / D_1 = e_o / D_1 = 0.3$, $L / D_1 = 3.5$, and $t_w = 1.4 \text{ mm}$

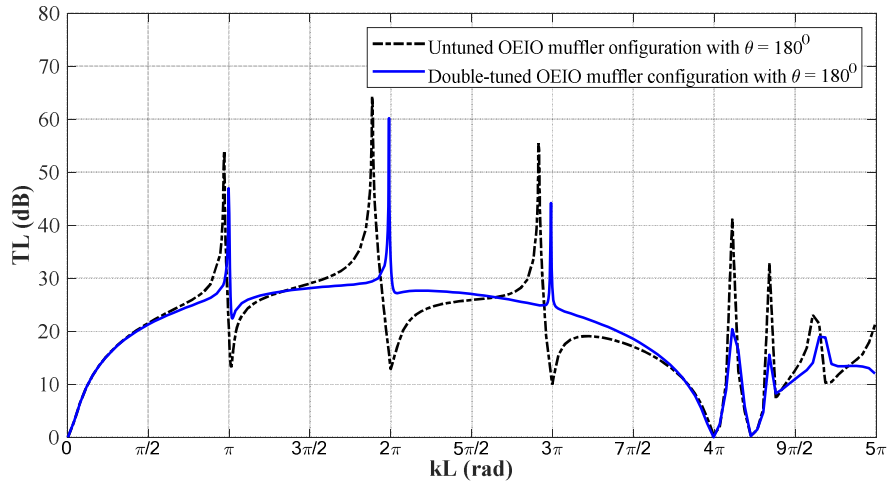


Fig. 5 — Comparison of the TL spectra of untuned and double-tuned SEIO muffler configurations with $\theta=180^0$, $d = 30 \text{ mm}$, $m = D_1 D_2 / d^2 = 16$, $AR = D_2 / D_1 = 0.4$, $e_i / D_1 = e_o / D_1 = 0.3$, $L / D_1 = 3.5$, and $t_w = 1.4 \text{ mm}$

It can be observed from Fig. 3 that for SEIO muffler configuration the 1st and 3rd TL spectrum peaks shift towards right whereas the 2nd peak shifts towards left. The crucial role of end-corrections of extended pipes on the TL spectra of mufflers can be appreciated by observing Figs. 3-5. In some frequency region the untuned muffler configurations have better performance as compared with their tuned muffler configuration counterparts. However, the real worry for muffler designers is the troughs appearing at $kL = \pi, 2\pi, 3\pi$. If the frequency range which gives significant noise contribution depends on the operating conditions of source, then double-tuned configurations would be always preferred. The end-corrections of both extended pipes of muffler configurations which TL spectra are shown in Figs. 3-5 are listed in Table 2.

Table 2—End-corrections of muffler configurations which TL spectra are shown in Figs. 3-5

Configuration	θ	$\delta_{0.5L}$	$\delta_{0.25L}$
SEIO	180^0	10.8 mm	10.2 mm
OEIO	0^0	11.6 mm	12.8 mm
OEIO	180^0	10.6 mm	10.2 mm

To compute the end-corrections, 3-D FEA is carried out with 0.5 Hz resolution around

the first peak and 1 Hz resolution around the second peak.

3. Non-dimensional parameters for the study

The value of the normalized end-corrections δ/d is something which is needed in terms of all non-dimensional geometrical parameters to double-tune any muffler configuration without doing the 3-D FEA. The generalized expressions of the non-dimensional end-corrections should be of the form given by Eqs. 3 and 4.

$$\frac{\delta_{0.5L}}{d} = f\left(\frac{D_1 D_2}{d^2}, \frac{D_2}{D_1}, \frac{L}{D_1}, \frac{t_w}{d}, \frac{e_{0.5L}}{D}, \frac{e_{0.25L}}{D}, \frac{\theta}{\pi}\right) \quad (3)$$

$$\frac{\delta_{0.25L}}{d} = f\left(\frac{D_1 D_2}{d^2}, \frac{D_2}{D_1}, \frac{L}{D_1}, \frac{t_w}{d}, \frac{e_{0.5L}}{D}, \frac{e_{0.25L}}{D}, \frac{\theta}{\pi}\right) \quad (4)$$

In the present study, the effect of t_w/d and θ/π is not considered for both SEIO and OEIO muffler configurations. However, for OEIO muffler configurations 3-D FEA are carried out for $\theta = 0^0$ and $\theta = 180^0$. The inlet/outlet pipes are of 30 mm diameter and 1.4 mm thickness for all configurations studied in the present study. The range of non-dimensional parameter values for which 3-D FEA is carried out are listed in Table 3.

Table 3—Range of the non-dimensional parameters taken in the 3D FEA to compute the end-corrections.

Parameter	SEIO	OEIO
$m = \frac{D_1 D_2}{d^2}$	12, 16, 20	12, 16, 20
$AR = D_2/D_1$	0.4, 0.5, 0.6, 0.7, 0.8, 0.9	0.4, 0.5, 0.6, 0.7, 0.8, 0.9
L/D_1	3.5, 4, 4.5	3.5, 4, 4.5
$e_{0.5L}/D_1$	0.15, 0.2, 0.25, 0.3	0.1, 0.2, 0.3
$e_{0.25L}/D_1$	0.15, 0.2, 0.25, 0.3	0.1, 0.2, 0.3
θ/π	1	0, 1

4. Parametric study on end-corrections for the SEIO elliptical chamber muffler with $\theta = 180^\circ$

The variation in $\delta_{0.5L}/d$ and $\delta_{0.25L}/d$ due to variation in AR , $e_{0.5L}/D_1$, $e_{0.25L}/D_1$ for $L/D_1 = 3.5$ and $m = D_1 D_2/d^2 = 16$ is shown in Table 4 for SEIO muffler with $\theta = 180^\circ$. It can be observed from Table 4 that the variation of $e_{0.25L}/D_1$ does not have effect on $\delta_{0.5L}/d$ for the frequency resolution is 0.5 Hz near the first TL peak. However, the increase in $e_{0.5L}/D_1$ decreases the $\delta_{0.25L}/d$. Figures 6 and 7, and Table 4 shows that the increase in an extended pipe’s offset increases that pipe’s end-correction. Figure 8 shows that there is a little decrement in $\delta_{0.25L}/d$ as there is an increment in $e_{0.5L}/D_1$ for constant $e_{0.25L}/D_1$.

It can be also observed from Table 4 that the increase in AR for particular pair of $(e_{0.5L}, e_{0.25L})$ decreases the values of $\delta_{0.5L}/d$ and $\delta_{0.25L}/d$. The effect of AR on the normalized end-corrections while other non-dimensional geometrical parameters are kept constant are shown in Fig. 9. It can be noticed from Table 4 that $\delta_{0.5L}/d \geq \delta_{0.25L}/d$ for $e_{0.5L}/D_1 \geq e_{0.25L}/D_1$ and $\delta_{0.25L}/d > \delta_{0.25L}/d$ for $e_{0.25L}/D_1 > e_{0.5L}/D_1$.

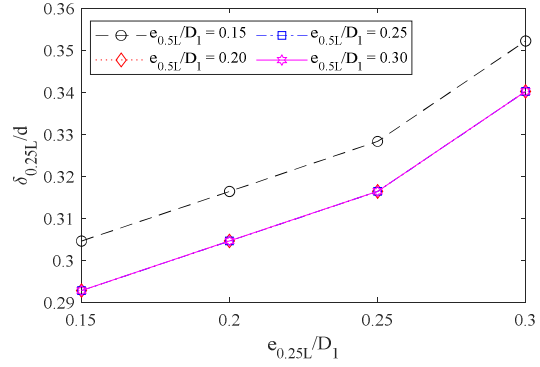


Fig. 6 — Effect of $e_{0.25L}/D_1$ on $\delta_{0.25L}/d$ of SEIO muffler with $\theta = 180^\circ$, $m = 16$, $AR = 0.4$, $L/D_1 = 3.5$ and $t_w = 1.4 \text{ mm}$

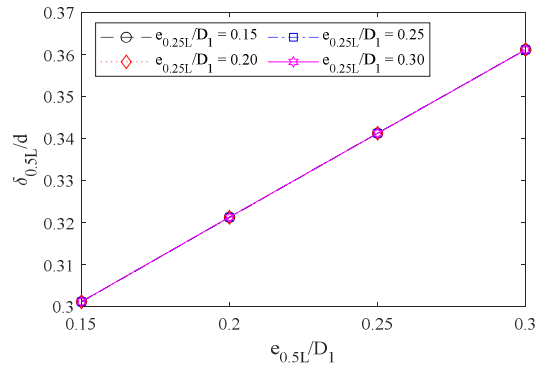


Fig.7 — Effect of $e_{0.5L}/D_1$ on $\delta_{0.5L}/d$ of SEIO muffler with $\theta = 180^\circ$, $m = 16$, $AR = 0.4$, $L/D_1 = 3.5$ and $t_w = 1.4 \text{ mm}$

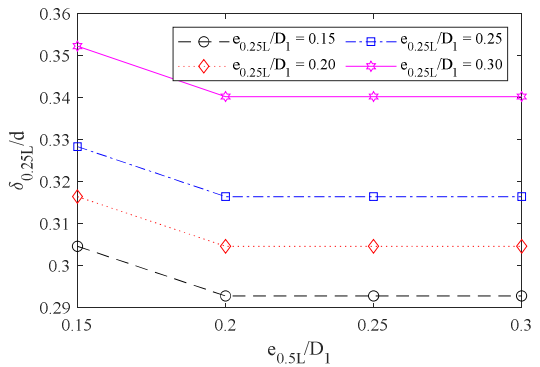


Fig.8 — Effect of $e_{0.5L}/D_1$ on $\delta_{0.25L}/d$ of SEIO muffler with $\theta = 180^\circ$, $m = 16$, $AR = 0.4$, $L/D_1 = 3.5$ and $t_w = 1.4 \text{ mm}$

Table 4— Effect of $e_{0.5L}/D_1$, $e_{0.25L}/D_1$ and AR on $\delta_{0.5L}/d$ and $\delta_{0.25L}/d$ for SEIO mufflers with $\theta = 180^\circ$, $L/D_1 = 3.5$ and $m = 16$

	$e_{0.5L}/D_1 = 0.15$	$e_{0.5L}/D_1 = 0.2$	$e_{0.5L}/D_1 = 0.25$	$e_{0.5L}/D_1 = 0.3$				
AR = 0.4								
	$\delta_{0.5L}/d$	$\delta_{0.25L}/d$	$\delta_{0.5L}/d$	$\delta_{0.25L}/d$	$\delta_{0.5L}/d$	$\delta_{0.25L}/d$	$\delta_{0.5L}/d$	$\delta_{0.25L}/d$
$e_{0.25L}/D_1 = 0.15$	0.3012	0.3046	0.3213	0.2928	0.3413	0.2928	0.3612	0.2928
$e_{0.25L}/D_1 = 0.2$	0.3012	0.3164	0.3213	0.3046	0.3413	0.3046	0.3612	0.3046
$e_{0.25L}/D_1 = 0.25$	0.3012	0.3283	0.3213	0.3164	0.3413	0.3164	0.3612	0.3164
$e_{0.25L}/D_1 = 0.3$	0.3012	0.3522	0.3213	0.3402	0.3413	0.3402	0.3612	0.3402
AR = 0.5								
$e_{0.25L}/D_1 = 0.15$	0.2970	0.2921	0.3129	0.2921	0.3289	0.2826	0.3447	0.2826
$e_{0.25L}/D_1 = 0.2$	0.2970	0.3016	0.3129	0.3016	0.3289	0.2921	0.3447	0.2921
$e_{0.25L}/D_1 = 0.25$	0.2970	0.3112	0.3129	0.3112	0.3289	0.3016	0.3447	0.3016
$e_{0.25L}/D_1 = 0.3$	0.2970	0.3401	0.3129	0.3304	0.3129	0.3304	0.3447	0.3208
AR = 0.6								
$e_{0.25L}/D_1 = 0.15$	0.2937	0.2908	0.3070	0.2828	0.3202	0.2828	0.3333	0.2828
$e_{0.25L}/D_1 = 0.2$	0.2937	0.2988	0.3070	0.2908	0.3202	0.2908	0.3333	0.2828
$e_{0.25L}/D_1 = 0.25$	0.2937	0.3069	0.3070	0.3069	0.3202	0.2988	0.3333	0.2988
$e_{0.25L}/D_1 = 0.3$	0.2937	0.3312	0.3070	0.3231	0.3202	0.3150	0.3333	0.3150
AR = 0.7								
$e_{0.25L}/D_1 = 0.15$	0.2914	0.2894	0.3027	0.2825	0.3139	0.2825	0.3364	0.2756
$e_{0.25L}/D_1 = 0.2$	0.2914	0.2963	0.3027	0.2894	0.3139	0.2825	0.3252	0.2825
$e_{0.25L}/D_1 = 0.25$	0.2914	0.3033	0.3027	0.2963	0.3139	0.2963	0.3252	0.2963
$e_{0.25L}/D_1 = 0.3$	0.2914	0.3243	0.2914	0.3173	0.3139	0.3103	0.3252	0.3103
AR = 0.8								
$e_{0.25L}/D_1 = 0.15$	0.2897	0.2940	0.2995	0.2818	0.3094	0.2818	0.3289	0.2757
$e_{0.25L}/D_1 = 0.2$	0.2897	0.2940	0.2995	0.2879	0.3094	0.2818	0.3289	0.2818
$e_{0.25L}/D_1 = 0.25$	0.2897	0.3002	0.2995	0.2940	0.3094	0.2940	0.3289	0.2940
$e_{0.25L}/D_1 = 0.3$	0.2897	0.3187	0.2995	0.3125	0.3094	0.3064	0.3289	0.3064
AR = 0.9								
$e_{0.25L}/D_1 = 0.15$	0.2882	0.2918	0.2970	0.2809	0.3056	0.2809	0.3229	0.2754
$e_{0.25L}/D_1 = 0.2$	0.2882	0.2918	0.2970	0.2863	0.3056	0.2809	0.3229	0.2809
$e_{0.25L}/D_1 = 0.25$	0.2882	0.3028	0.2970	0.2973	0.3056	0.2918	0.3229	0.2918
$e_{0.25L}/D_1 = 0.3$	0.2882	0.3195	0.2970	0.3139	0.3056	0.3084	0.3229	0.3028

It can be observed from Figs. 9-11 that the AR is the significant parameter which affects the δ/d , however, the effect of

variation of m and L/D_1 on δ/d is neither well behaved nor significant.

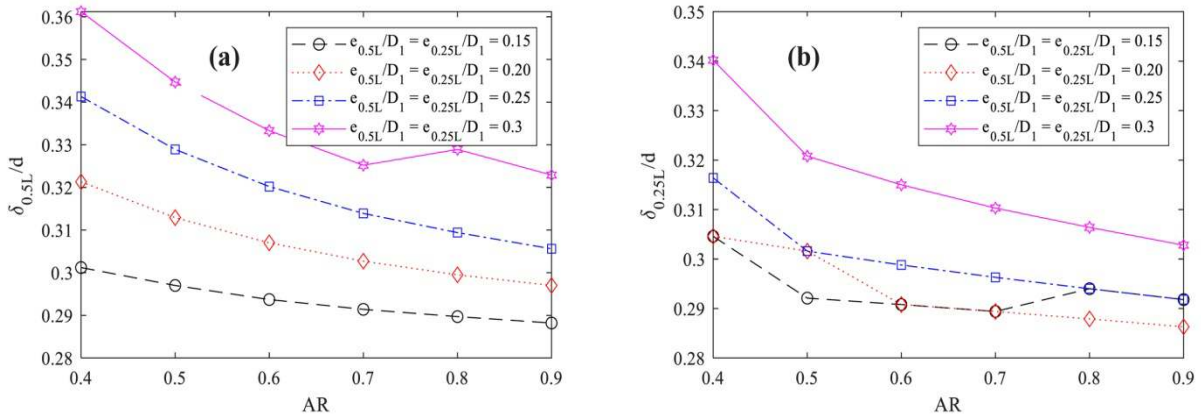


Fig.9 — Effect of AR on δ/d of SEIO muffler with $\theta=180^\circ$, $m = 16$, $L/D_1=3.5$ and $t_w=1.4$ mm: (a) $\delta_{0.5L}/d$ and (b) $\delta_{0.25L}/d$

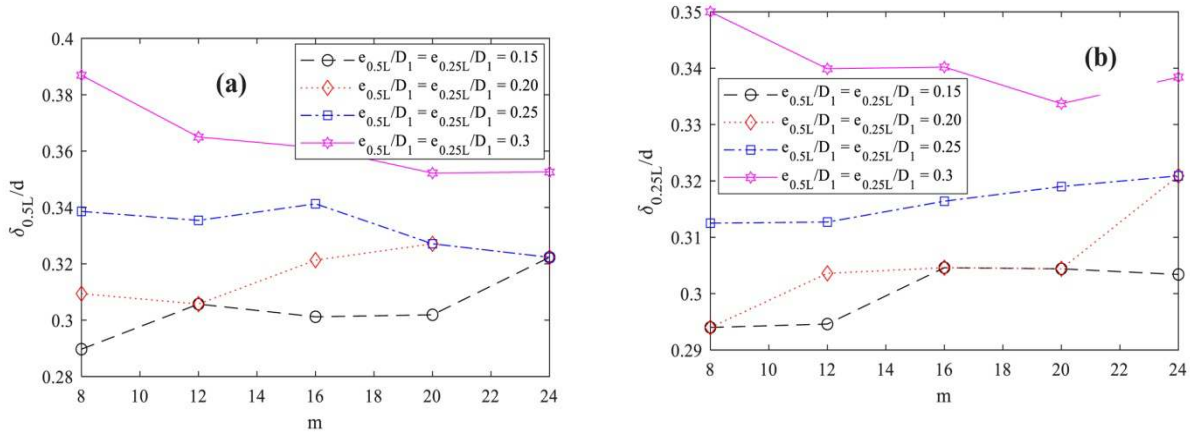


Fig.10 — Effect of m on δ/d of SEIO muffler with $\theta=180^\circ$, $AR = 0.4$, $L/D_1=3.5$ and $t_w=1.4$ mm: (a) $\delta_{0.5L}/d$ and (b) $\delta_{0.25L}/d$

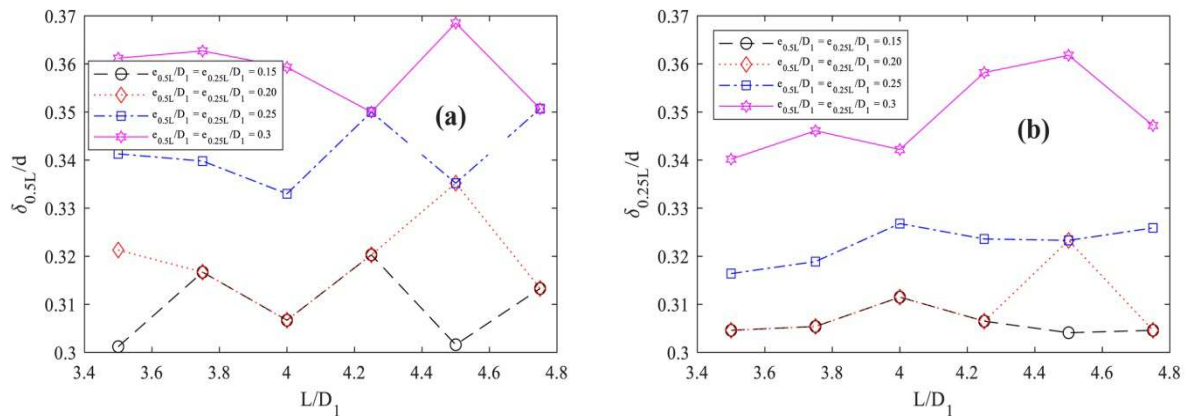


Fig.11 — Effect of L/D_1 on δ/d of SEIO muffler with $\theta=180^\circ$, $AR = 0.4$, $m = 16$ and $t_w=1.4$ mm: (a) $\delta_{0.5L}/d$ and (b) $\delta_{0.25L}/d$

5. Parametric study on end-corrections for the OEIO elliptical chamber muffler with $\theta = 0^\circ$ and $\theta = 180^\circ$

The variation in $\delta_{0.5L}/d$ and $\delta_{0.25L}/d$ due to variation in AR , $e_{0.5L}/D_1$, $e_{0.25L}/D_1$ for $L/D_1 = 3.5$ and $m = D_1 D_2 / d^2 = 16$ is shown in Table 5 and Table 6 for SEIO muffler with $\theta = 0^\circ$ and $\theta = 180^\circ$, respectively. It can be observed that the trends of variation of SEIO muffler with $\theta = 180^\circ$ in Table 4 and OEIO mufflers with $\theta = 180^\circ$ in Table 6 are similar. However, the variation in $\delta_{0.25L}/d$ with variation in $e_{0.25L}/D_1$ for fixed $e_{0.5L}/D_1$

and other parameters is more for OEIO muffler configurations with $\theta = 0^\circ$ as compared with SEIO and OEIO muffler configurations with $\theta = 180^\circ$. In fact, if all other parameters are kept fixed then the normalized end-corrections values are more for $\theta = 0^\circ$ than $\theta = 180^\circ$. The similar trends were reported by Kumar *et al.*² for circular chamber mufflers.

Table 5— Effect of $e_{0.5L}/D_1$, $e_{0.25L}/D_1$ and AR on $\delta_{0.5L}/d$ and $\delta_{0.25L}/d$ for OEIO mufflers with $\theta = 0^\circ$, $L/D_1 = 3.5$ and $m = 16$

	$e_{0.5L}/D_1 = 0.1$		$e_{0.5L}/D_1 = 0.2$		$e_{0.5L}/D_1 = 0.3$	
	AR = 0.4					
	$\delta_{0.5L}/d$	$\delta_{0.25L}/d$	$\delta_{0.5L}/d$	$\delta_{0.25L}/d$	$\delta_{0.5L}/d$	$\delta_{0.25L}/d$
$e_{0.25L}/D_1 = 0.1$	0.2952	0.3047	0.3176	0.3165	0.3627	0.3165
$e_{0.25L}/D_1 = 0.2$	0.2952	0.3404	0.3176	0.3404	0.3627	0.3524
$e_{0.25L}/D_1 = 0.3$	0.2952	0.3886	0.3176	0.4130	0.3854	0.4253
	AR = 0.5					
$e_{0.25L}/D_1 = 0.1$	0.2978	0.3017	0.3159	0.3017	0.3522	0.3113
$e_{0.25L}/D_1 = 0.2$	0.2978	0.3305	0.3159	0.3402	0.3522	0.3402
$e_{0.25L}/D_1 = 0.3$	0.2978	0.3695	0.3159	0.3892	0.3522	0.4090
	AR = 0.6					
$e_{0.25L}/D_1 = 0.1$	0.2844	0.2989	0.2995	0.2989	0.3146	0.3070
$e_{0.25L}/D_1 = 0.2$	0.2844	0.3232	0.3146	0.3313	0.3450	0.3395
$e_{0.25L}/D_1 = 0.3$	0.2995	0.3641	0.3146	0.3807	0.3450	0.3974
	AR = 0.7					
$e_{0.25L}/D_1 = 0.1$	0.2874	0.2964	0.3004	0.2964	0.3397	0.3034
$e_{0.25L}/D_1 = 0.2$	0.2874	0.3174	0.3004	0.3315	0.3397	0.3385
$e_{0.25L}/D_1 = 0.3$	0.2874	0.3599	0.3135	0.3743	0.3528	0.3887
	AR = 0.8					
$e_{0.25L}/D_1 = 0.1$	0.2894	0.2941	0.3009	0.3003	0.3353	0.3003
$e_{0.25L}/D_1 = 0.2$	0.2894	0.3126	0.3009	0.3250	0.3353	0.3375
$e_{0.25L}/D_1 = 0.3$	0.2894	0.3564	0.3123	0.3754	0.3469	0.3882
	AR = 0.9					
$e_{0.25L}/D_1 = 0.1$	0.2907	0.2919	0.3010	0.2974	0.3317	0.3029
$e_{0.25L}/D_1 = 0.2$	0.2907	0.3140	0.3010	0.3251	0.3421	0.3363
$e_{0.25L}/D_1 = 0.3$	0.2907	0.3533	0.3112	0.3703	0.3421	0.3875

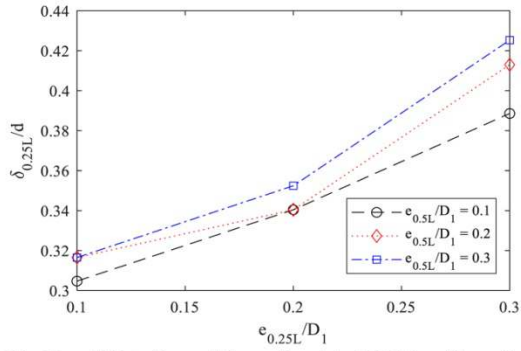


Fig. 12 — Effect of $e_{0.25L}/D_1$ on $\delta_{0.25L}/d$ of OEIO muffler with $\theta = 0^\circ$, $m = 16$, $AR = 0.4$, $L/D_1 = 3.5$ and $t_w = 1.4$ mm

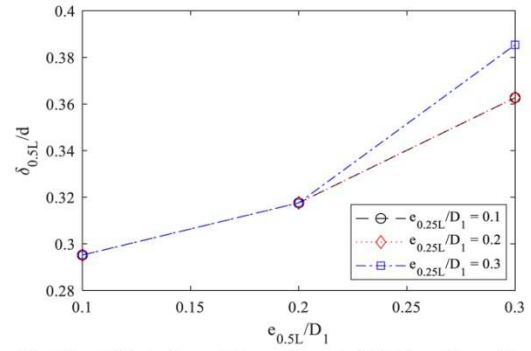


Fig. 13 — Effect of $e_{0.5L}/D_1$ on $\delta_{0.5L}/d$ of SEIO muffler with $\theta = 0^\circ$, $m = 16$, $AR = 0.4$, $L/D_1 = 3.5$ and $t_w = 1.4$ mm

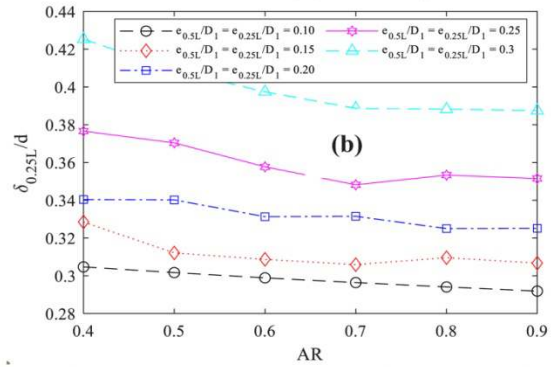
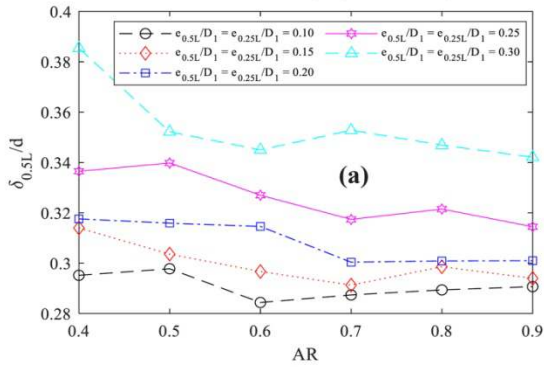


Fig. 14 — Effect of AR on δ/d of OEIO muffler with $\theta = 0^\circ$, $m = 16$, $L/D_1 = 3.5$ and $t_w = 1.4$ mm: (a) $\delta_{0.5L}/d$ and (b) $\delta_{0.25L}/d$

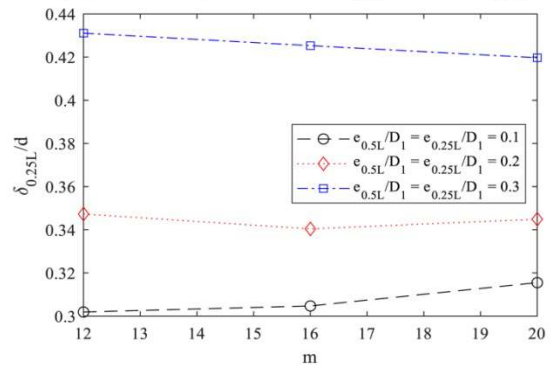
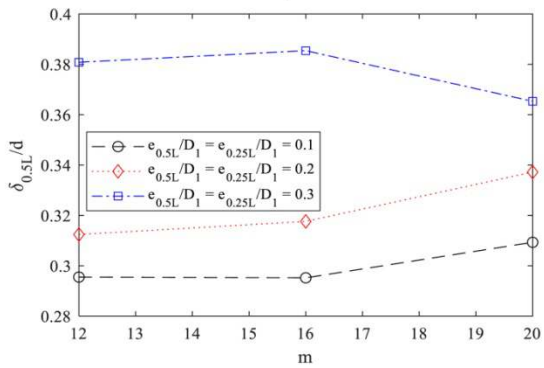


Fig. 15 — Effect of m on δ/d of OEIO muffler with $\theta = 0^\circ$, $AR = 0.4$, $L/D_1 = 3.5$ and $t_w = 1.4$ mm: (a) $\delta_{0.5L}/d$ and (b) $\delta_{0.25L}/d$

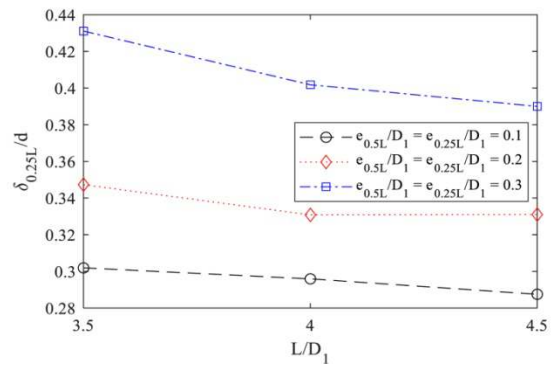
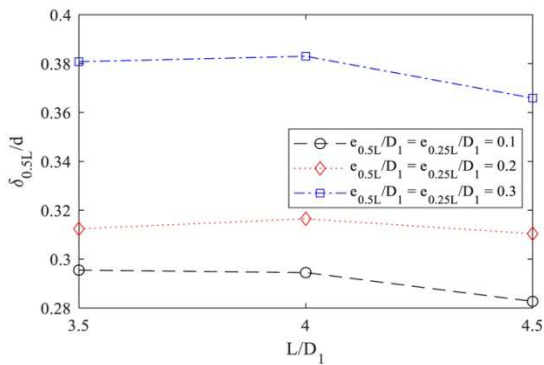


Fig. 16 — Effect of L/D_1 on δ/d of OEIO muffler with $\theta = 0^\circ$, $AR = 0.4$, $m = 16$ and $t_w = 1.4$ mm: (a) $\delta_{0.5L}/d$ and (b) $\delta_{0.25L}/d$

Figures 12 and 13 show the effect of $e_{0.25L}/D_1$ on $\delta_{0.25L}/d$, and $e_{0.5L}/D_1$ on $\delta_{0.5L}/d$, respectively. The effect of AR on the end-corrections on $\delta_{0.5L}/d$ and $\delta_{0.25L}/d$ for OEIO mufflers with $\theta = 0^\circ$ are shown in Figs. 14(a) and 14(b), respectively.

6. Conclusions

The effect of aspect ratio of ellipse, offset of the extended pipes and the angle between the inlet and outlet port are found to have significant effect on the end-corrections of pipes. A more extensive

including more simulations with different angles between inlet and outlet port is required to give the parametric expressions for end-corrections as was done in Ref.² for circular chamber mufflers.

Table 6— Effect of $e_{0.5L}/D_1$, $e_{0.25L}/D_1$ and AR on $\delta_{0.5L}/d$ and $\delta_{0.25L}/d$ for OEIO mufflers with $\theta = 180^\circ$, $L/D_1 = 3.5$ and $m = 16$

	$e_{0.5L}/D_1 = 0.1$		$e_{0.5L}/D_1 = 0.2$		$e_{0.5L}/D_1 = 0.3$	
	AR = 0.4					
	$\delta_{0.5L}/d$	$\delta_{0.25L}/d$	$\delta_{0.5L}/d$	$\delta_{0.25L}/d$	$\delta_{0.5L}/d$	$\delta_{0.25L}/d$
$e_{0.25L}/D_1 = 0.1$	0.2876	0.2929	0.3100	0.2929	0.3550	0.2929
$e_{0.25L}/D_1 = 0.2$	0.2876	0.3165	0.3100	0.3047	0.3550	0.3047
$e_{0.25L}/D_1 = 0.3$	0.2876	0.3524	0.3100	0.3404	0.3550	0.3404
	AR = 0.5					
$e_{0.25L}/D_1 = 0.1$	0.2910	0.2922	0.3090	0.2826	0.3452	0.2826
$e_{0.25L}/D_1 = 0.2$	0.2910	0.3017	0.3090	0.2922	0.3452	0.2922
$e_{0.25L}/D_1 = 0.3$	0.2910	0.3402	0.3090	0.3305	0.3452	0.3209
	AR = 0.6					
$e_{0.25L}/D_1 = 0.1$	0.2934	0.2829	0.3085	0.2829	0.3389	0.2749
$e_{0.25L}/D_1 = 0.2$	0.2934	0.2989	0.2934	0.2909	0.3237	0.2829
$e_{0.25L}/D_1 = 0.3$	0.2784	0.3313	0.2934	0.3150	0.3237	0.3070
	AR = 0.7					
$e_{0.25L}/D_1 = 0.1$	0.2820	0.2825	0.2950	0.2756	0.3342	0.2756
$e_{0.25L}/D_1 = 0.2$	0.2820	0.2964	0.2950	0.2825	0.3211	0.2756
$e_{0.25L}/D_1 = 0.3$	0.2820	0.3244	0.2950	0.3104	0.3211	0.3034
	AR = 0.8					
$e_{0.25L}/D_1 = 0.1$	0.2844	0.2819	0.2958	0.2758	0.3188	0.2697
$e_{0.25L}/D_1 = 0.2$	0.2844	0.2941	0.2958	0.2819	0.3188	0.2758
$e_{0.25L}/D_1 = 0.3$	0.2844	0.3188	0.2958	0.3064	0.3188	0.3003
	AR = 0.9					
$e_{0.25L}/D_1 = 0.1$	0.2865	0.2809	0.2967	0.2755	0.3274	0.2700
$e_{0.25L}/D_1 = 0.2$	0.2865	0.2919	0.2865	0.2809	0.3171	0.2755
$e_{0.25L}/D_1 = 0.3$	0.2763	0.3196	0.2865	0.3029	0.3171	0.2974

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