Prediction of narrow tube periodic structure transmission loss using the Transfer matrix method

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Abstract

Narrow-tube periodic structure panels have applications in multiple fields like aerospace, the automobile industry, and other engineering applications due to their high stiffness-to-weight ratio. These structures were investigated in terms of strength and manufacturing in the existing literature and limited research for acoustic performance prediction. However, the acoustic performance of these panels in terms of absorption coefficient and transmission loss needs to be studied mainly at lower frequencies. The present manuscript discusses the prediction of transmission loss using the transfer matrix method. The structure proposed in this paper is a sandwich panel with three layers in total. The top and bottom layers are made with polypropylene tubes arranged periodically and separated by a thin paper membrane layer. The transfer matrix of each layer is developed. Acoustic wave propagation in a narrow tube is calculated using equivalent speed and density. The calculated transfer matrix relates the upstream acoustic pressure and velocity with the downstream acoustic variables. The transfer matrices of the layers are cascaded to obtain the total transfer matrix and calculate the transmission loss as a function of the frequency. The calculated results are compared to the measured transmission loss using the impedance tube. The analytical and experimental results are found to be in good agreement.

Keywords: Periodic structure, narrow tube, transmission loss, transfer matrix method, impedance tube,

1. Introduction

Periodic cellular structures are used widely in engineering applications due to their high stiffness-to-weight ratio. The individual cells in this structure form an array of hollow tubes which gives it high compression strength while being very lightweight. These structures are therefore widely used in aerospace, automobile, and structural applications that require high strength and low weight. While the structural properties of these panels are extensively studied, their acoustic properties are not that well explored. Having a good understanding of the acoustic properties of systems which will have good acoustic properties along with the required structural properties.These structures can also be used to absorb low-frequency noise which traditionally requires very thick panels to absorb. With traditional acoustic panels, to absorb a sound wave of wavelength λ, then it is required an acoustic panel of at least λ/4 thicknessto get the maximum absorption. Therefore, when the frequency become sufficiently low, it becomes very impractical to use traditional panels because their thickness becomes very high, and they will also become bulky and costly to

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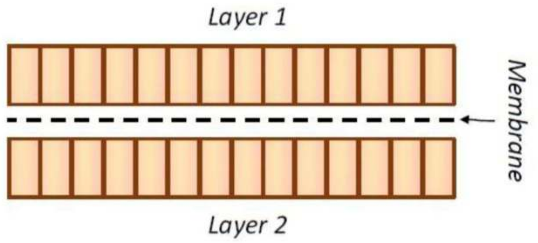
manufacture. So,it is proposed to design acoustic panels using periodic cellular structures and tuning them to absorb sound in these low-frequency ranges while being lightweight and thin. However, the acoustic transmission loss of these panels needs to be evaluated along with the absorption coefficient. The present manuscript discusses transmission loss prediction at lower frequencies.

Hannink Marieke [1] studied the acoustic properties of tube resonators and developed mathematical models of different complexities to predict the TL. He observed that the length of tube resonators, the porosity of the panel, and the mass of the panel were the three most important parameters that influenced the insulating properties of panels with tube resonators. Satish Kumar [2] used both the sandwich theory for wave propagation and the classic orthotropic plate theory to evaluate the transmission loss of honeycomb panels. Deepak Akiwate [3] studied the absorption behaviour of periodic cellular structures using the impedance tube method. The propagation constant and characteristic acoustic impedance for louvers with narrow Helmholtz resonators were studied by Sakamoto, Shuichi [4]. D. J. Mead, [5] studied the propagation of waves in continuous periodic structures. J. Allard and N. Atalla [6] modelled the multi-layered porous materials using the transfer matrix method. The formulation to obtain the transmission loss from the transfer matrix of panels was given by M. P. Norton and D. G. Karczub [7].

2. Methodology

The transfer matrix method is ananalytical method which helps toanalyze the propagation of sound waves through a multi-layered and complex structure. This method cascades the individual subsystem or component transfer matrices to get the overall transfer matrix of the complex system. This method is the perfect tool for analyzing the acoustic properties of panels and walls made of periodic structures, which are generally used in the form of sandwiched panels. For validating the mathematical model, the TL of sandwiched honeycomb panels were measured using impedance tube method and compared with the analytical results. The panels had two layers of periodic cellular structure separated by a thin flexible paper membrane



2.1 Transfer matrix of the periodic cells

Consider a layer of periodic tubes with rigid walls, as shown in Figure 2. Let's say that the acoustic pressure and particle velocity at plane 1 and plane 2 immediately outside the layer are correspondingly denoted as P1, V1, and P4, V4, respectively. Also, the acoustic pressure and particle velocity within the tubes at corresponding planes are denoted by the values P2, V2, and P3, V3, respectively.

Assuming pressure and velocity boundary conditions

$$P_1 = P_2, \quad (1a)$$

$$v_1 = \frac{nA_2}{A_1} v_2 = (A.R.)v_2 \quad (1b)$$

$$P_3 = P_4 \quad (2a)$$

$$, v_4 = \frac{nA_3}{A_4} v_3 = (A.R.)v_3 \quad (2b)$$

The transfer matrix can be written as,

$$TM_{core} = \begin{bmatrix} \cos(kL) & j\,Z_c \sin(kL)/(A.R.) \\ j\,Z_c (A.R.)\sin(kL) & \cos(kL) \end{bmatrix} \quad (3)$$

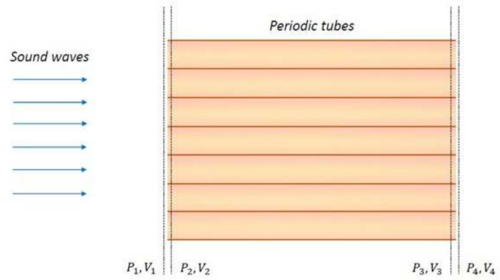


Figure 2: Acoustic wave propagation through a honeycomb core

$$k = \omega\sqrt{\frac{\rho_{eff}}{K_{eff}}}, \qquad Z_c = \sqrt{\frac{K_{eff}}{\rho_{eff}}} \qquad (4)$$

$\rho_{eff} = \rho_0\left(1+\frac{\sigma\phi}{j\omega\rho_0}G_s(s)\right)$	$K = \frac{\gamma P_0}{[\gamma-[(\gamma-1)F(B^2w)]}$
$G_c(s) = \frac{\frac{-s}{4}\sqrt{-j}\frac{J_1(s\sqrt{-j})}{J_0(s\sqrt{-j})}}{1-\frac{2}{s\sqrt{-j}}\frac{J_1(s\sqrt{-j})}{J_0(s\sqrt{-j})}}$	$s = \bar{c}\sqrt{\frac{8\omega\rho_0}{\sigma\varphi}}$
$F(B^2\omega) = \frac{1}{\left[1+\frac{\sigma\varphi}{jB^2\omega\rho_0}\right]G_c(Bs)}$	$\bar{c} = \frac{1}{\bar{r}}\sqrt{\frac{8\eta}{\sigma\varphi}}$

When the sound waves travel through the cells, narrow tube effects also come into play, andthe consequences of temperature gradients and viscosity cannot be ignored. Both the density and the speed of sound are no longer fixed values but instead change depending on the frequency. The smaller the cells, the more the narrow tube effects will dominate. In smaller tubes, the thickness of the boundary layer formed on the tube walls becomes significant compared to the tube dimensions. Which causes a higher percentage of sound waves to pass through the boundary layer, which increases the acoustic impedance of the periodic cells. The viscous and thermal conduction losses at the boundary of a narrow tube are the primary mechanisms responsible for acoustic dissipation. This is because acoustic energy causes heat to diffuse throughout the tube. It is described in general form by the narrow tube theory, according to which the influence of shape can be realized by adjusting the shape-dependent component$\bar{c}$. Both flow resistivityσ and porosityϕ can alter depending on the form of the tube they are measured in. Both the viscous and thermal effects were taken into consideration independently and included, in the modified complex density and the complex bulk modulus.

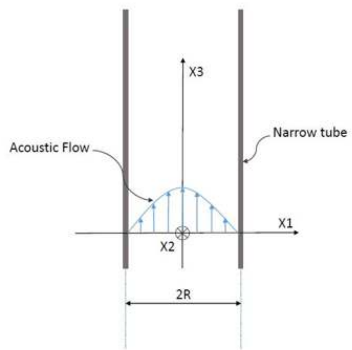


Figure 2: Velocity of air flowing through narrow tubes

To account for the narrow tube effects, the values of k and Z_c are modified.
Where,

$\bar{r}$ → Hydraulic radius of the cell
ρ_0 → density of air
γ → ratio of specific heats of air
$A.R.$ → Area ratio
$\sigma\varphi = \frac{7.5\eta}{\bar{r}^2}$ (for hexagonal cell)
ω → Frequency of sound in radians/sec
L → thickness of the core layer
η → Dynamic viscosity of air
$\bar{c} = 1.0328$ (for hexagonal cell)
J_0 → Bessel function of zeroth order
J_1 → Bessel function of first order

2.2 Transfer Matrix of the Membrane

Given the driving pressure Δp, initial tension T, and surface density (ρ_m), a governing equation for the displacement of a circular membrane (ξ_m), expressed as a function of radial position (r_m) from the membrane's center, is given as follows:

$$T\nabla^2\xi_m(r_m) + 2j\omega\eta_m\nabla^2\xi_m(r_m) + \omega^2\rho_m\xi_m(r_m) = -\Delta p \quad (3)$$

Where,

η_m – Damping ratio of the material,

Δp – Driving pressure,

ω – Frequency (radians)

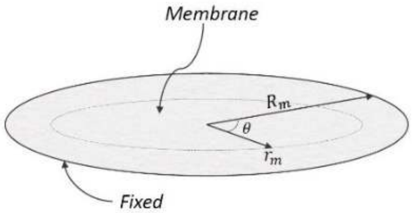


Figure 3: Paper membrane

Solving equation (5) subjected to the boundary condition $\xi_m(r_m = R_m) = 0$. Gives the membrane displacement Differentiate the equation of displacement to get the equation for membrane velocity. The equation for space averaged velocity of membrane ($\bar{v}_{mem}$) can be obtained by integrating the velocity equation over the surface and normalizing it with area is as follows

$$\bar{v}_{mem} = j\omega\frac{\Delta p}{\omega^2\rho_m}\left[\frac{2}{K_{mem}R_m}\frac{J_1(K_{mem}R_m)}{J_0(K_{mem}R_m)} - 1\right] \quad (4)$$

The acoustic impedance can be written as,

$$Z_{mem} = -\frac{\Delta p}{\bar{v}_{mem}} = -j\omega\rho_m\left[\frac{2}{K_{mem}R_m}\frac{J_1(K_{mem}R_m)}{J_0(K_{mem}R_m)} - 1\right]^{-1} \quad (5)$$

$$TM_{mem} = \begin{bmatrix}1 & Z_{mem}\\ 0 & 1\end{bmatrix} \quad (6)$$

The transfer matrix can be formed from acoustic impedance as,

2.3 Transfer matrix of the entire sandwiched panel

To get the combined transfer matrix, multiply the transfer matrices of each layer in the sequence in which they are arranged from the incident side to the transmitted side. In the present configuration as shown in Figure 1, a membrane layer is sandwiched between two honeycomb core layers, so the transfer matrix can be obtained as:

$$TM_{SHP} = TM_{core}TM_{mem}TM_{core} \quad (7)$$

Substitute equation (3) and equation (8) inequation (9) to get the transfer matrix for the entire panel.

2.4 Calculating the transmission loss from the transfer matrix

The transmission loss of the panel can be calculated from the transfer matrix.

Consider the overall transfer matrix to be in the form of:

$$\begin{bmatrix}T_{11} & T_{12}\\ T_{21} & T_{22}\end{bmatrix}$$

The transmission loss in dB can be calculated as:

$$TL = 20log_{10}\left|\frac{T_{11} + {T_{12}}/{Z_0} + T_{21}Z_0 + T_{22}}{2}\right| \quad (8)$$

Where,

$Z_0 = \rho_0 c_0$

$\rho_0 \rightarrow$ Density of air,

$c_0 \rightarrow$ Speed of sound in air

3. Results and discussion

For validating the analytical model, the TL of sandwiched honeycomb panels was measured using the impedance tube method and compared with the analytical results. The panels had two layers of a periodic cellular honeycomb structure made of polypropylene material, separated by a thin flexible paper membrane. The details of the two tested panels are given in Table 1.

Table 1: Dimensions of the sandwich panels

Panel	Panel 1	Panel 2
Core thickness	17 mm	17 mm
Cell size	6 mm	6.3 mm
Cell wall thickness	0.5 mm	0.7 mm
Membrane thickness	0.2 mm	0.2 mm
Open area ratio	0.7562	0.80196

A MATLAB code was written based on the formulation discussed in the previous section, to get the transmission loss of the sandwiched honeycomb panel in the desired frequency range. The results obtained from the code are compared with the experimental results to verify the mathematical model. The readings were taken from the two sandwiched panels, the details of which are mentioned in Table 1.

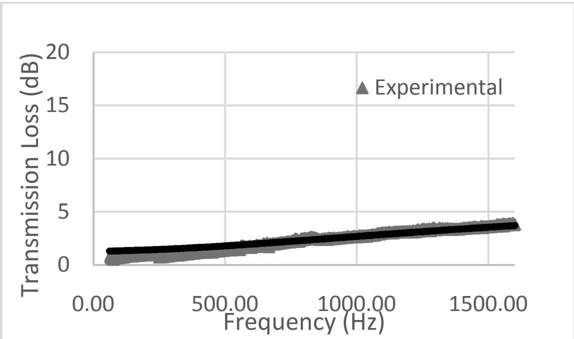


Figure 4. Comparison of transmission loss prediction results for panel 1

It is observed from Figures 4 and 5 that there is a good correlation between the analytical and the experimental results, which validatesthe analytical method. The TL of the panel is very low as there isn't enough mass that the sound waves have to pass through to get to the other side of the panel. The TL is also increasing with increasing frequency.

From the model, it can be observed that panels with smaller cells will have better acoustic properties as the narrow tube effects will play a larger role in them. Narrow tubes increase the acoustic impedance mismatch which increases the transmission loss of the panels. Smaller cells also mean that there is more material in the panel, hence the mass of the panel will be higher, which will also contribute to the increase in TL of the panel which can be explained by the mass law, which states that for every doubling of mass of a panel, its TL increases by 6dB. In the model developed, the cell walls were rigid, further improvements could be made in the TL by using cells with flexible walls, which would further increase the impedance mismatch.

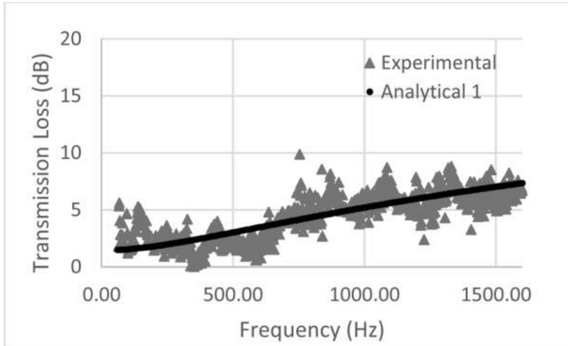


Figure 5. Comparison of transmission loss prediction results for panel 2

4. Conclusion

In this paper, a methodology is proposed to estimate the transmission loss of periodic cells using the transfer matrix method. The narrow tube effects are incorporated in the calculated transfer matrices of cells. When the sound waves incident on the panel, the wave experiences narrow tube effects and a sudden change in the acoustic impedance. The higher the impedance mismatch and the larger the TL of sound, the narrower the tubes. The developed methodology was validated by measuring transmission losses experimentally using the impedance tube method.

The measurements were taken using sandwiched honeycomb panels, which are used most frequently in engineering applications. The experimental and analytical values are found to be in good agreement with each other.

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