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Systematic Analysis of Oxygen Diffusion Problem having Local Fractional Derivative

Ravi Shanker Dubey[a], Manvendra Narayan Mishra[a],* and Pranay Goswami[b]

[a]*Amity University Rajasthan, Jaipur-302030, India*
[b]*Dr. B.R. Ambedkar University, Delhi, India*

Abstract : In this paper, we examine another dependable calculation in light of the nearby partial homotopy perturbation Sumudu transform procedure (LFHPSTM). We additionally characterize a numerical model which depicts the dissemination of oxygen in retaining tissue, and found the mathematical arrangements of its partial differential condition utilizing LFHPSTM. The above characterize method gives the outcomes with practically no change of the situation into discrete partners or forcing prohibitive presumptions and is totally liberated from adjust blunders.

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Introduction

Dispersion of oxygen within engrossing cell was initially concentrated by Crank and Gupta [1]. Whenever, oxygen is permitted to disperse within the medium, little part of oxygen is consumed by channel also ingestion of oxygen at outer layer of channel is kept up with consistent. This period of the issue go on till a consistent condition is attained in that the air doesn't enter any more is fixed so no air moves inside or outside, the channel keeps on retaining accessible air currently in that and, consequently, the limit in the consistent condition go about subside about fixed plane. Crank and Gupta [2] additionally utilized static space matrix mobile within limit and vital interjections are carried out along with each of two block interpolant or multinomials. Noble proposed rehashed structural region [3], the hotness equilibrium integral procedure explained by Reynolds and Dalton [4], an orthogonal cluster to find solution of partial differential equations of dispersion of air in engrossing cell explained by Liapis et al. [5]. Two numeric procedures to solve oxygen dispersion problem were given by GÜlkaÇ [6]. Mitchell gave exact use of integral procedure [7]. For more explanation, visit [8]– [17].

Derivative is the commonly used concept in mathematics. It indicates the rate of variation of function. Also, it is thoughtful to explain several real world incidents. Subsequently, the researchers found few tedious issue of society to get then researchers established fractional differentiation (look [9]– [13]). The idea of fractional calculus is of greater importance in several fields and also salient for formulating social issues (look [14]– [17]). Several unseen features of actual world situations from many disciplines were evolved by applying fractional PDE. Fractional differential operators (FDOs) are being used effectively to dispense numeric analytical devices with capabilities for use in mathematics, science, physics and mechanics [1–4]. Getting precise and effective procedures to solve FDEs are being a dynamic testing agreement. In last 10 years, many systematic and numeric procedures have been given to solve linear and nonlinear fractional differential equations, such as Adomian decomposition

**Corresponding author:(E-mail: manvendra.mishra22187@gmail.com)*

method (ADM) (see [5]-[7]), homotopy perturbation method (HPM) (see [8]-[9]) , homotopy analysis method (HAM) (see [11]), were strongly investigated for solving various families of fractional differential equations. A semi-numerical technique, called differential transform method (DTM), was first invented by Zhou (see [12]) and its leading utilizations are resolved for both linear and nonlinear initial value issues. Elsiad (see [13]), considered DTM coupling with the Adomian polynomials. Recently, local fractional calculus has come out as a major field to explain the tedious phenomena. Local fractional differentiations are normally very hard to resolve scientifically hence it is mandatory to get an effective nearby result, and for the same, many analytical and numerical approaches were formulated (see [14]- [20]).

Roused and persuaded by the continuous examination around here and broad uses of local fractional conditions, our suggestion is to use LFHPSTM for solution of local fractional system of Oxygen Diffusion related to local fractional extremities value conditions. LFHPSTM is the combination of old style homotopy perturbation method (HPM) (see [8]- [9]) and local fractional Sumudu transformation procedure.

Prime purpose of the paper is to identify a semi-numerical method, n-dimensional extended differential transform (n-DEDT), in context to get non-differentiable result of local fractional differential equation. The article is structured as mentioned below:

Segment 1 deals with the foundation part. In part 2, we provide conceptions related to local fractional differential operators (LFDOs). In segment 3, fractional air dispersion system is discussed. Segment four deals with LFHPSTM. Segment 5 is having numeric applications. We concluded our observations in last section.

Local Fractional Integral and Differential Operators ([18]-[22])

In current segment, we audit essential hypothesis of local fractional math, that has been used in the paper.

Definition 1. Consider the relationship $\left|x-x_0\right|<\delta$ where $\varepsilon,\delta>0$ and $\varepsilon\in R$ we allow function $\Phi(x)\in C_k(a,b)$,

$$\left|\Phi(x)-\Phi(x_0)\right|<\varepsilon^k, 0<k\leq 1. \quad (2.1)$$

Definition2. Take the gap [a,b] with $(t_i,t_{i+1}), i=0,1,2,...,n-1;\ t_0=a\, and\, t_n=b,$ with $\Delta t_i=t_{i+1}-t_i,\ \Delta t=\max(\Delta t_0,\Delta t_1,...),$ a division of gap. Then, local fractional integral is explained below:

$$\begin{aligned} {}_aI_b^{(k)}\Phi(x)&=\frac{1}{\Gamma(1+k)}\int_a^b\Phi(t)(dt)^k \\ &=\frac{1}{\Gamma(1+k)}\lim_{\Delta t\to 0}\sum_{i=0}^{n-1}\Phi(t_i)(\Delta t_i)^k, \end{aligned} \quad (2.2)$$

Definition 3. Assume the function $\Phi(x)$, satisfy state in Equation (2.1), then, at that point, the converse recipe of Equation (2.2) is characterized as follows:

$$D^k\Phi(x_0)=\frac{d^k\Phi(x_0)}{dx^k}=\lim_{x\to x_0}\frac{\Delta^k\left(\Phi(x)-\Phi(x_0)\right)}{\left(x-x_0\right)^k} \quad (2.3)$$

where

$$\Delta^k\left(\Phi(x)-\Phi(x_0)\right)\cong\Gamma(1+k)\left[\Phi(x)-\Phi(x_0)\right] \quad (2.4)$$

The definition of local fractional derivative, employed in the article, is defined below:

$$\frac{d^k}{dx^k}\frac{x^{kn}}{\Gamma(1+nk)}=\frac{x^{(n-1)k}}{\Gamma(1+(n-1)k)},\ n\in N. \quad (2.5)$$

Oxygen Diffusion Fractional System

System of air dispersion was introduced by Crank and Gupta. Air dispersion issue has two numerical phases. At primary phase, steady state happens when oxygen is infused into again from within or beyond the tissue then the cell exterior is splitted. In subsequent phase, cells begin to ingest infused oxygen. The mobile limit issue brought about by this stage. Point of this interaction is to get an equilibrium state and deciding the time-subordinate varying limit location. Details of time-partial of oxygen dispersion issue can be found (see [23]-[26]). Ulkac discussed the relative study among two numeric procedures for oxygen diffusion issue.

Liapis et al defined a system of air dispersion in absorbing cells. Mitchell found the use of integral method used in diffusion of air in absorbing cell. Alkahtani discussed the fractional air dispersion issue with nonsingular kernel.

Take the oxygen diffusion problem mentioned below:

$$\frac{\partial^{\alpha}\left(c(x,t)\right)}{\partial t^{\alpha}} = c_{xx} - 1; \quad x,t \in \phi \tag{3.1}$$

along with starting and extreme conditions defined as

$$c(x,0) = \frac{(1-x)^2}{2}, \quad 0 \le x \le 1, \tag{3.2}$$

$$\frac{\partial c}{\partial x} = 0, \; x = 0, t \ge 0 \tag{3.3}$$

$$c = \frac{\partial c}{\partial x} = 0, \; x = s(t), t \ge 0, with \;\; s(0) = 1. \tag{3.4}$$

here $0 < \alpha \le 1$.

Study of LFHPSTM ([27]- [32])

Watugala suggested and fostered the Sumudu transform and Belgacem et al. gave a portion of its significant properties. Belgacem explained Sumudu transform to address fractional differential conditions. Belgacem explored the utilizations of Sumudu transform in Bessel function. Belgacem and dubey have examined Fractional outspread dispersion condition utilizing Sumudu transformation. Bhavna Gupta et al. (see [33]) had utilized the Sumudu transform for solving summarized fractional kinetic conditions. Belgacem in 2006 presented further Sumudu properties. Goswami et al. showed significance of sumudu transform in tackling generalized time-partial message condition. Bulut et al. obtained the insightful arrangements of a few fractional differential conditions by utilizing Sumudu transform strategy. The HPSTM explored by Atangana et al. in tackling space-time partial (see [34]- [40]) Fokker-Planck condition. Srivastava et al. introduced the local fractional Sumudu transform which is characterized beneath:

$$\frac{d^k}{dx^k}\frac{x^{kn}}{\Gamma(1+nk)} = \frac{x^{(n-1)k}}{\Gamma\left(1+(n-1)k\right)}, \; n \in N. \tag{4.1}$$

$$LFS_k\left\{\Phi(x)\right\} = \Phi_k(z) \tag{4.2}$$

$$= \frac{1}{\Gamma(1+k)}\int_0^{\infty} E_k\left(-z^{-k}x^k\right)\Phi(x)\, z^{-k} \tag{4.3}$$

its inverse formula is given below

$$LFS^{-1}{}_k\left\{\Phi_k(z)\right\} = \Phi(x), \quad 0 < k \le 1 \tag{4.4}$$

In context to setup the fundamental plan of LFHPSTM,
suppose the linear differential equation along with local
fractional differential operator mentioned below-

$$L_k v(x,t) + R_k v(x,t) = h(x,t) \tag{4.5}$$

here L_k represents the linear local fractional differential
coefficient, R_k represents the prevailing linear operator and $h(x,t)$ denote the source expression. Now,

$$\begin{aligned}\psi_k(x,z) &= v(x,0) + z^k v^k(x,0) + z^{2k}v^{2k}(x,0) + \\ &\dots + z^{(n-1)k}v^{(n-1)k}(x,0) - z^{nk}LFS_k\left[R_k v(x,t)\right] + \\ &\qquad z^{nk}LFS_k\left[R_k h(x,t)\right]\end{aligned} \tag{4.6}$$

Now, using inverse local fractional Sumudu transform in Equation (4.6),

$$\begin{aligned}v(x,t) &= v(x,0) + \frac{t^k}{\Gamma(1+k)}v^k(x,0) + \\ &\frac{t^{2k}}{\Gamma(1+2k)}v^{2k}(x,0) + \dots \\ &+ \frac{t^{(n-1)k}}{\Gamma\left(1+(n-1)k\right)}v^{(n-1)k}(x,0) \\ &- LFS_k^{-1}\left[z^{kn}LFS_k\left(R_k v(x,t)\right)\right] \\ &+ LFS_k^{-1}\left[z^{kn}LFS_k\left(R_k h(x,t)\right)\right]\end{aligned} \tag{4.7}$$

Further, applying the HPM,

$$v(x,t)=\sum_{m=0}^{\infty}p^{m}v_{m}(x,t). \tag{4.8}$$

Putting Equation (4.6) in Equation (4.5), we obtain:

$$\sum_{m=0}^{\infty}p^{m}v_{m}(x,t)=v(x,0)+\frac{t^{k}}{\Gamma(1+k)}v^{k}(x,0)+\frac{t^{2k}}{\Gamma(1+2k)}v^{2k}(x,0)+\dots+\frac{t^{(n-1)k}}{\Gamma(1+(n-1)k)}v^{(n-1)k}(x,0)-LFS_{k}^{-1}\left[z^{kn}LFS_{k}\left(R_{k}\sum_{m=0}^{\infty}p^{m}v_{m}(x,t)\right)\right]+LFS_{k}^{-1}\left[z^{kn}LFS_{k}\left(R_{k}h(x,t)\right)\right] \tag{4.9}$$

that is the combination of local fractional Sumudu transformation procedure along with HPM. Contrasting the multiples of similar exponent of p, we have

$$p^{0}:v_{0}(x,t)=v(x,0)+\frac{t^{k}}{\Gamma(1+k)}v^{k}(x,0)+\frac{t^{2k}}{\Gamma(1+2k)}v^{2k}(x,0)+\dots+\frac{t^{(n-1)k}}{\Gamma(1+(n-1)k)}v^{(n-1)k}(x,0)+LFS_{k}^{-1}\left[z^{kn}LFS_{k}\left(R_{k}h(x,t)\right)\right] \tag{4.10}$$

$$p^{1}:v_{1}(x,t)=-LFS_{k}^{-1}\left[z^{kn}LFS_{k}\left(R_{k}v_{0}(x,t)\right)\right] \tag{4.11}$$

$$p^{2}:v_{2}(x,t)=-LFS_{k}^{-1}\left[z^{kn}LFS_{k}\left(R_{k}v_{1}(x,t)\right)\right] \tag{4.12}$$

$$\vdots$$

and so on.

Hence, the solution is given as

$$v(x,t)=\lim_{m\to\infty}\sum_{m=0}^{M}v_{m}(x,t).$$

5. Applications

5.1 Case-I At first, we look into the local fractional Tricomi equations given below

$$\frac{\partial^{\alpha}\left(c(x,t)\right)}{\partial t^{\alpha}}=c_{xx}-1;\quad x,t\in\phi \tag{5.1}$$

along with starting and extreme constraints mentioned below-

$$c(x,0)=\frac{(1-x)^{2}}{2},\quad 0\le x\le 1, \tag{5.2}$$

$$\frac{\partial c}{\partial x}=0,\ x=0,t\ge 0 \tag{5.3}$$

$$c=\frac{\partial c}{\partial x}=0,\ x=s(t),t\ge 0,\mathit{with}\ \ s(0)=1. \tag{5.4}$$

here $0<\alpha\le 1.$

Further, using local fractional Sumudu transformation in Equation (5.1), we have

$$\bar{c}(x,z)=c(x,0)-z^{\alpha}\left[LFS\left\{\left(\frac{\partial^{2}c}{\partial x^{2}}\right)-1\right\}\right] \tag{5.5}$$

$$\bar{c}(x,z)=0.5(1-x)^{2}-z^{\alpha}\left[LFS\left\{\left(\frac{\partial^{2}c}{\partial x^{2}}\right)-1\right\}\right] \tag{5.6}$$

Then applying inverse local fractional Sumudu transformation in Equation (5.6), we obtain

$$c(x,t)=0.5(1-x)^2-$$
$$pLFS^{-1}\left\{z^{\alpha}\left[LFS\left\{\left(\frac{\partial^2 c}{\partial x^2}\right)-1\right\}\right]\right\} \quad (5.7)$$

Now using HPM we get

$$\sum_{m=0}^{\infty} p^m c_m(x,t)=0.5(1-x)^2-$$
$$pLFS^{-1}\left\{z^{\alpha}\left[LFS\left\{\left(\frac{\partial^2}{\partial x^2}\left(\sum_{m=0}^{\infty} p^m c_m(x,t)\right)\right)-1\right\}\right]\right\} \quad (5.8)$$

Looking at similar exponent of p, we have accompanying parts of series arrangement

$$p^0=c_0(x,t)=0.5(1-x)^2,$$

$$p^1=c_1(x,t)=2\frac{t^{\alpha}}{\Gamma(\alpha+1)},$$

$$p^2=c_2(x,t)=0,$$

$$\vdots$$

and so on. Hence, outcome is acquired below

$$c(x,t)=\lim_{m\to\infty}\sum_{m=0}^{M} c_n(x,t).$$

$$c(x,t)=0.5(1-x)^2+2\frac{t^{\alpha}}{\Gamma(\alpha+1)}. \quad (5.9)$$

5.2 **Case-II** Now suppose an expression for position of the mobile boundary s(t). So,

$$\frac{\partial^{\alpha}(s(x,t))}{\partial t^{\alpha}}=-c_x\big|_{x=s(t)}; \quad (5.10)$$

with the starting and extreme constraints given below

$$s(0)=1, \quad (5.11)$$

$$c=\frac{\partial c}{\partial x}=0,\ x=s(t),t\ge 0. \quad (5.12)$$

here $0<\alpha\le 1$.
Using local fractional Sumudu transformation in equation (5.10),

$$\bar{s}(x,z)=s(x,0)+z^{\alpha}\left[LFS\left(\frac{\partial c}{\partial x}\right)\right] \quad (5.13)$$

$$\bar{s}(x,z)=1+z^{\alpha}\left[LFS\left(\frac{\partial c}{\partial x}\right)\right] \quad (5.14)$$

Now, applying inverse local fractional Sumudu transformation in equation (5.14) implies

$$s(x,t)=1+pLFS^{-1}\left\{z^{\alpha}\left[LFS\left(\frac{\partial c}{\partial x}\right)\right]\right\} \quad (5.15)$$

Now using HPM we get

$$\sum_{m=0}^{\infty} p^m s_m(x,t)=1+$$
$$pLFS^{-1}\left\{z^{\alpha}\left[LFS\left\{\frac{\partial}{\partial x}\left(\sum_{m=0}^{\infty} p^m c_m(x,t)\right)\right\}\right]\right\} \quad (5.16)$$

Looking at similar exponent of p, we get the accompanying parts of series arrangement

$$p^0=s_0(x,t)=1,$$

$$p^1=s_1(x,t)=2\frac{t^{\alpha}}{\Gamma(\alpha+1)},$$

$$p^2=v_2(x,t)=0,$$

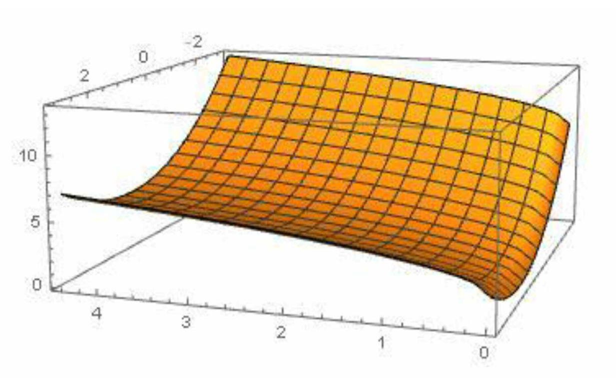
$$\vdots$$

and so on. Hence, the result is expressed as

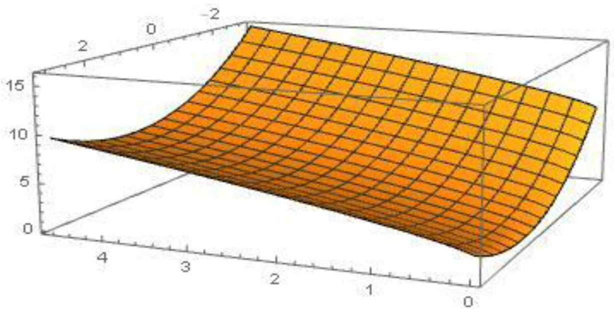
$$c(x,t)=\lim_{m\to\infty}\sum_{m=0}^{M} c_n(x,t). \quad (5.17)$$

$$c(x,t)=1+2\frac{t^{\alpha}}{\Gamma(\alpha+1)}. \quad (5.18)$$

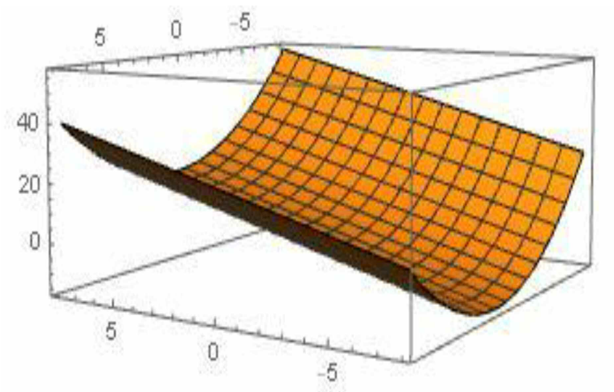
The figures, showing the outcomes for distinct values of , given by expression (5.9) are as follows-



For $\alpha = 0.5$

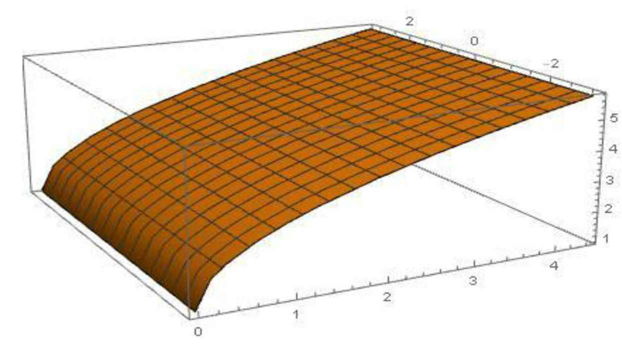


For $\alpha = 0.8$

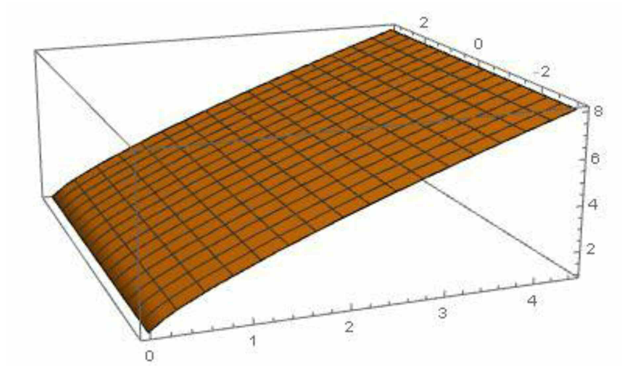


For $\alpha = 1$

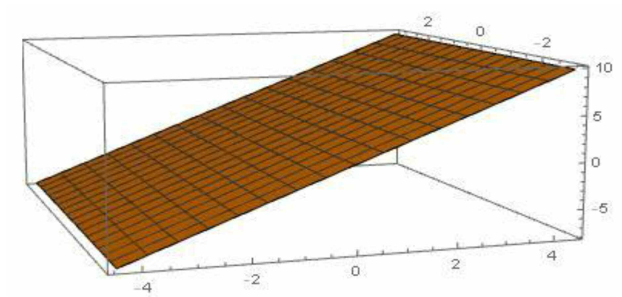
The figures, showing the outcomes for distinct values of , given by expression (5.18) are as follows-



For $\alpha = 0.5$



For $\alpha = 0.8$



For $\alpha = 1$

The graphs drawn here (of both cases) show the concentration distribution of oxygen at various time stage and at different values of . α

Conclusion

In current article, we used LFHPSTM to get result of the local fractional system of Oxygen Diffusion condition relating to local fractional subsidiary extremities conditions. We get the arrangement with non-differential expressions by using this methodology. Outcomes indicate that presented strategy is exceptionally productive in utilizing to tackle different sorts of local fractional differential conditions. Thus, the presented technique is an integral asset for settling local fractional linear conditions of actual significance. Diagrams drawn here (of the two cases) show the concentration circulation of oxygen at different time stage and at various upsides of .

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